FPT: $f(k)\, n^{O(1)}$

NP: $A \to B$

$A \leq B$

Lem: $B \in FPT$ u $A \xrightarrow[FPT]{} B \Rightarrow A \in FPT$

Definition 13.1 (Parameterized reduction). Let $A, B \subseteq \Sigma^* \times \mathbb{N}$ be two parameterized problems. A *parameterized reduction* from A to B is an algorithm that, given an instance (x, k) of A, outputs an instance (x', k') of B such that

1. (x, k) is a yes-instance of A if and only if (x', k') is a yes-instance of B,
2. $k' \leq g(k)$ for some computable function g, and
3. the running time is $f(k) \cdot |x|^{\mathcal{O}(1)}$ for some computable function f.

A

$(x, k) \rightarrow (x', k')$

$B \in FPT \Rightarrow h(k') \cdot |x'|^{c_2}$ $\leq$

$|x'| \leq f(k)|x|^{c_1}$

$k' \leq g(k)$

$\leq h(g(k)) (f(k)|x|^{c_1})^{c_2}$

$= h'(k)$

k-IS $\to$ k-Clique $\quad$ k-IS $\leq_p$ k-clique

$(G, k) \to (\bar{G}, k)$ $\quad$ k-IS $\leq_{FPT}$ k-clique

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k-IS $\to$ k-VC

$(G, k) \to (G, \underbrace{n-k}_{k'})$

k-IS $\leq_p$ k-VC

$Clique_{log}$ (G, k), $k \le \log n$

$Clique \le_{FPT} Clique_{log}$ $\quad 2 \cdot 2^k n$

$(G, k) \longrightarrow (\underbrace{G + \boxed{2^k}}, k)$

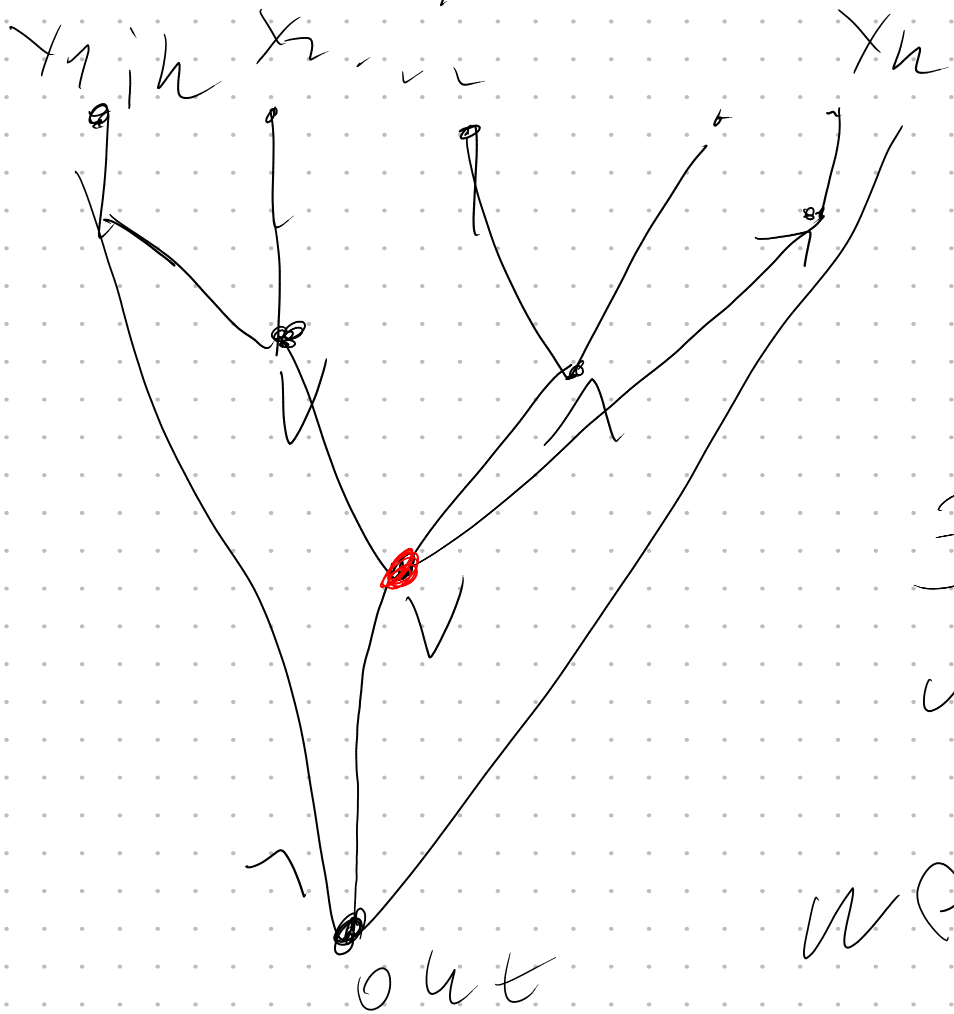
изолированные

$Clique_{log}$ $\in TIME(n^{O(\log n)})$

$\boxed{2^{O(\log^2 n)}}$

$A \leq_{FPT} B \leq_{FPT} C \implies A \leq_{FPT} C$

W – иерархия



gate v – большой $\Rightarrow \deg(v) > 2$

$\exists\, x_1, \ldots, x_n \in \{0,1\}$
и $\sum x_i = k$

weft

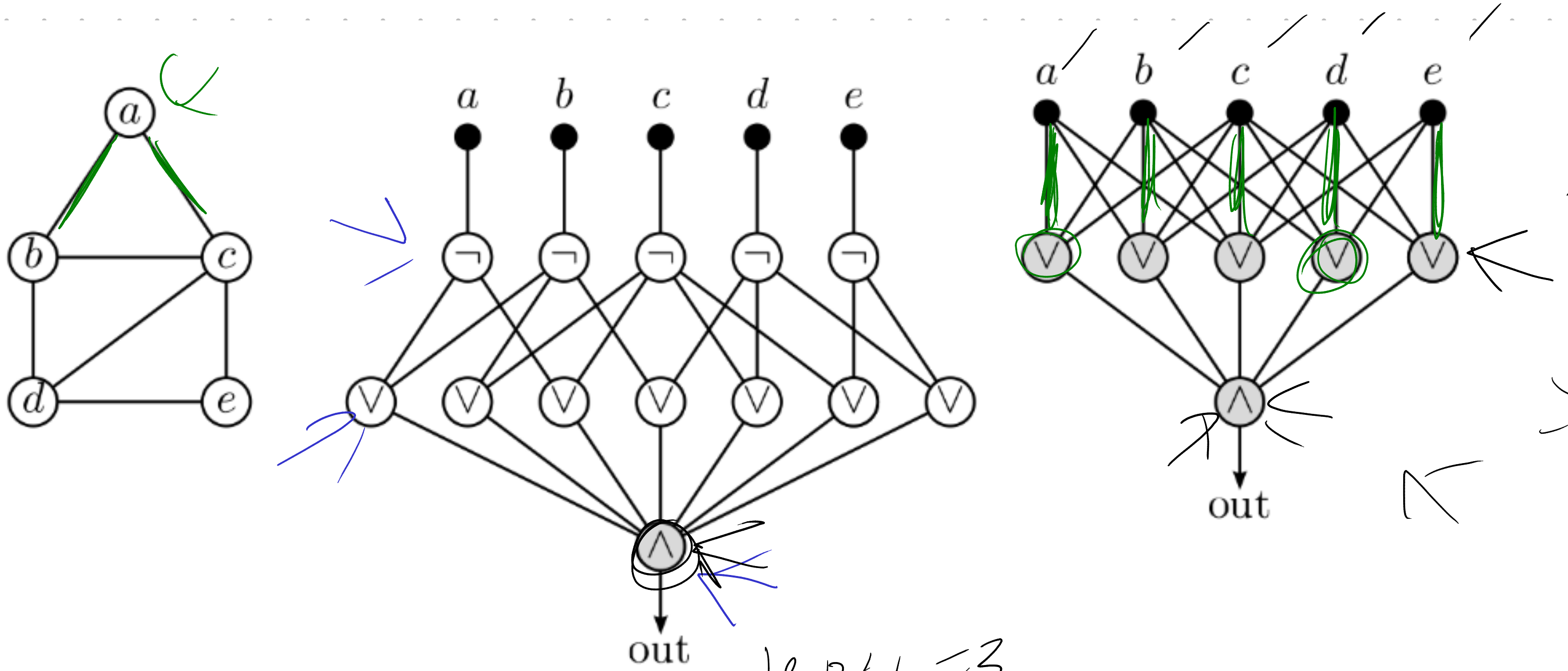
$C_{t,d} = \{ \underset{\text{схема}}{H} \mid \text{depth}(H) \leq d,\ \text{weft}(H) \leq t \}$

W – иерархия

$t \geq 1$ $\quad P \in W[t] \iff$

$\exists\ A \in C_{t,d}$, $d \geq 1$ $\quad P \leq_{fpt} A$

n



(a) (b) (c)

depth = 3
weft(1)

k-DS

$\Rightarrow$ k-DS $\in$ W[2]

IS, k $\Leftrightarrow$ VC, n-k

$\Rightarrow$ IS $\in$ W[1]

IS – W[1]-полная

$FPT \subseteq W[1]$

W

$FPT = \bigcup W[t]$

$A \in FPT \implies A \leq_{FPT} X$

$FP \cdot T \subseteq W[0]$ $\quad c \in C_{o,d}$

$FPT \supseteq W[0]$ $\quad n^k = (2^d)^k$

$FPT = W[0]$

t-нормализованная формы

$$\neg(x_1 \vee \overline{x_3} \vee \overline{x_5}) \wedge (x_1 \vee x_2) \wedge (x_1 \vee \overline{x_2})$$

2-normalized

Δ_0, Γ_0 — мн-во литералов

Δ_t — $\Gamma_{t-1} \vee \Gamma_{t-1} \vee \dots \vee \Gamma_{t-1}$

Γ_t — $\Delta_{t-1} \wedge \Delta_{t-1} \wedge \dots \wedge \Delta_{t-1}$

t-normalized form

t-монотонная нормализованная форма

Weighted Max-SAT

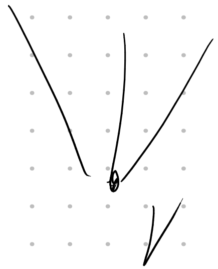
$t \geq 2$

1. $t \geq 0$, следующие $W[t]$-полны:
   - Weighted t-normalized SAT.
   - Weighted monotone t-norm. SAT
   - Weighted monotone $(t+1)$-norm. SAT

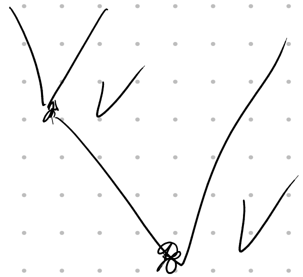
2. $t \geq 1$:
   - Weighted t-normalized SAT
   - Weighted antimonotone t-norm. SAT
   - Weighted antimonotone $(t+1)$-norm. SAT

dps > 2

dps > c



$\Longleftrightarrow$



$H \in FPT$

$c^{k} n^{O(1)} \leftarrow$

$\|$

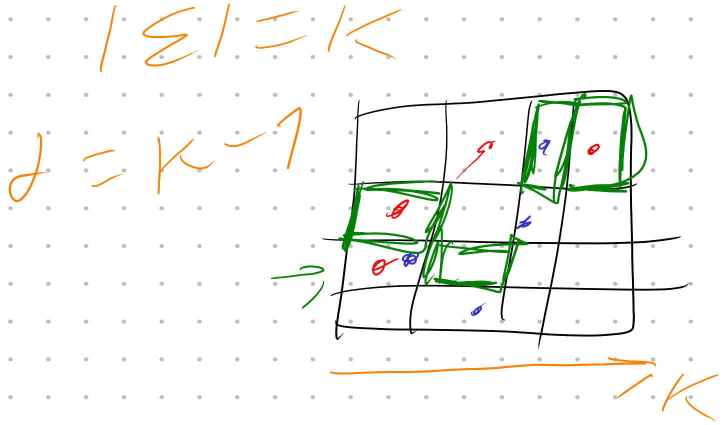
$2^{2^{2^{k}}} n^{O(1)} \leftarrow$

$k^{k} n^{O(1)} \Rightarrow 2^{k} n^{O(1)}$

ETH: 3-SAT $\notin T(2^{o(n)})$

$k \times k$ Hitting Set

$S_1, \dots, S_m$



$\exists ?\, S : S \cap S_i \neq \emptyset$ u $|S \cap column(i)| = 1$ $\forall i$

ETH $\Longrightarrow$ $k \times k$ HS $\notin T(2^{o(k \log k)} m^{O(1)})$

$|S_i \cap column(i)| \le 1$

Closest String

$S_1, \dots, S_t \in \Sigma^*$, $|S_i| = L$

$d \in \mathbb{N}$

$\exists S \in \Sigma^* : |S| = L : \mathrm{dist}(S, S_i) \le d \quad \forall i$

$CS \in T(\cancel{d}^{\,d} |\Sigma|^{O(1)})$

$ETH \Rightarrow CS \notin T(2^{o(d \log \max(d, |\Sigma|))} |\Sigma|^{O(1)})$

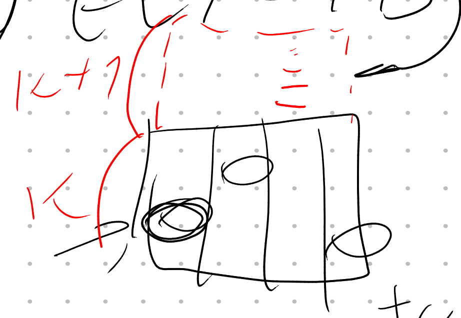
$S_1, \dots, S_m$ — instance $k \times k$ H-S

$|\Sigma| = 2k+1$

$L = k$

$d = k-1$

$S_x \to \{ S_{x,y} \mid y \in [k+1] \}$



$$S_{x,y}(i) = \begin{cases} S_x(i) & \text{elan } S_x \cap \text{column}(i) \neq \emptyset \\ y+k, & \text{elan } S_x \cap \text{column}(i) = \emptyset \end{cases}$$

$\rho : [k] \to [k]$

$(\underline{\rho(1)}, \underline{\rho(2)}, \dots, \underline{\rho(k)})\ f$

$\cap S_i \neq \emptyset \quad \forall i$

$\exists$ a S ; $S \cap S_{x,y}$ $\forall x, y$

$|S| = k$

$\exists\, y' \in [k+1]$: $S \cap (y'+k) = \emptyset$

$S \cap S_{x,y'} \neq \emptyset$ $\forall x$

S_x

$$p(i) = \begin{cases} s(i), & \text{om } s(i) \le k \\ 1, & \text{ellers} \end{cases}$$

$2^{O(d \log d)} \, |I|^{O(1)}$ und $2^{O(d \log |E|)} \, |I|^{O(1)}$

$d = k - 1$

$|S| = 2k + 1$ $\Rightarrow$ $\underline{k \times k \text{ HSGT}(2^{O(k \log k)} \, |I|^{O(1)})}$

XP : $TIME(n^{f(k)})$

$FPT \subseteq XP$

k-Coloring $\in XP$? , eunga

3-Coloring $\in T(n^{f(3)}) \Rightarrow$
$P = NP$

$FPT \subsetneq XP$