INCONSISTENT PLANNING: GRAPH MODIFICATION

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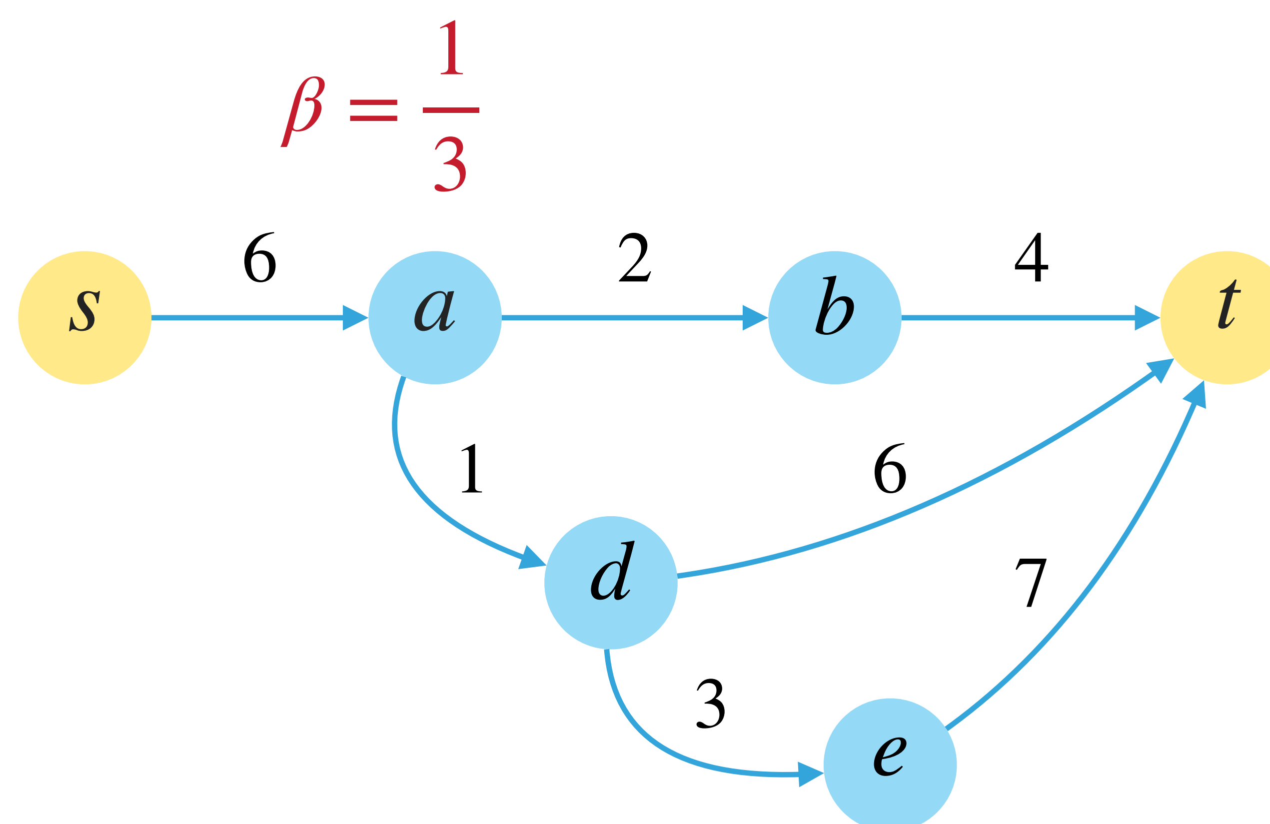
PRESENT-BIAS PLANNING

- Behavioral Economic
- Study the impact of the gap between the anticipated costs of future actions and their real costs.
- Time-inconsistent planning: procrastination, abandonment, etc.
- **Akerlof (1991):** Graph theoretical model, where the cost of an action in the future is assumed to be β times smaller than its actual cost, for some $\beta < 1$.

KLEINBERG–OREN'S MODEL (EC 2014)

5-tuple $M = (G, w, s, t, \beta)$, where:

- $G = (V(G), E(G))$ – DAG
- $w : E(G) \to \mathbb{N}$ – cost-function
- $s \in V(G)$ – start vertex
- $t \in V(G)$ – target vertex
- $\beta \leq 1$ – agent's present-bias parameter.

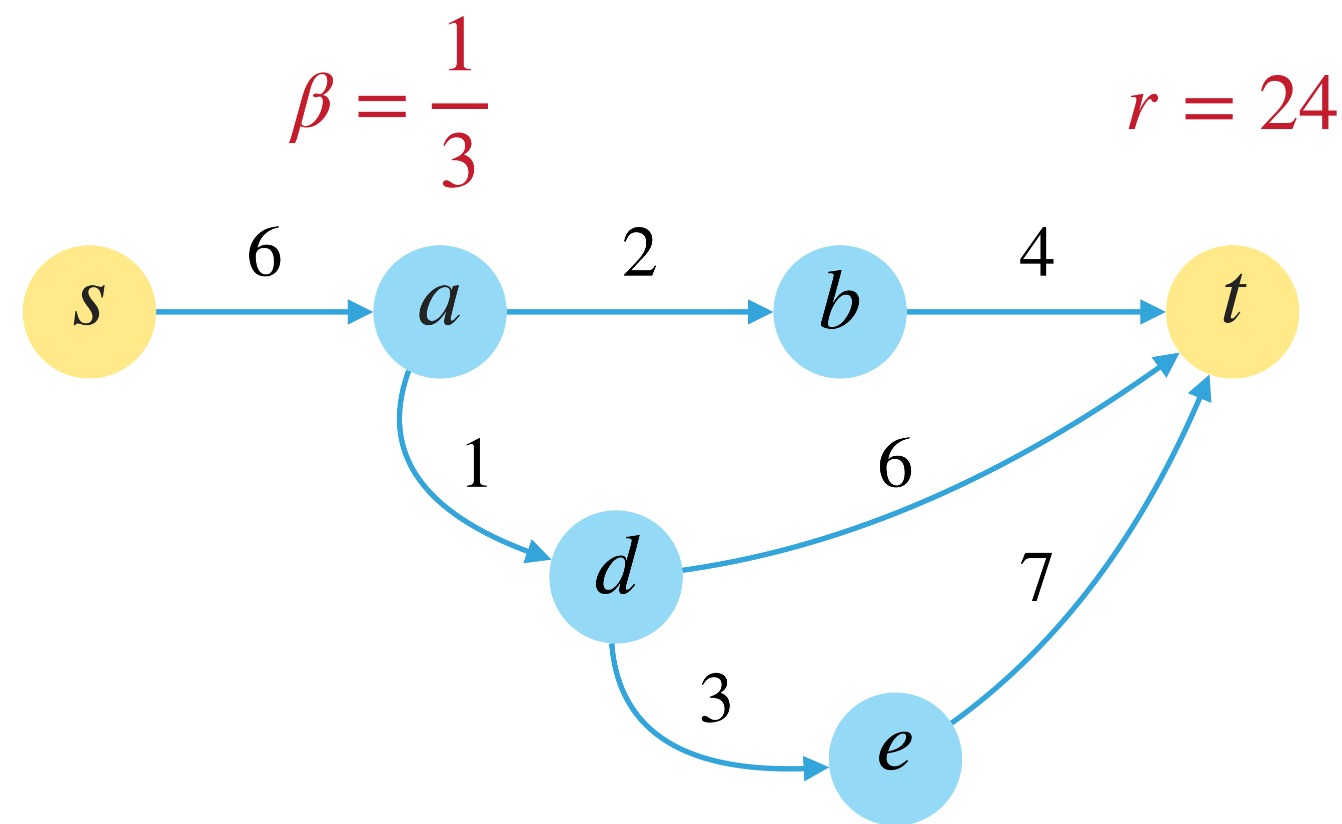


In vertex v agent evaluates a path $P \subseteq G$ with edges $e_1, e_2, \ldots, e_p$ to cost $\zeta_M(P) = w(e_1) + \beta \cdot \sum_{i=2}^{p} w(e_i)$.

MODEL WITH REWARD

6-tuple $M = (G, w, s, t, \beta, r)$, where:

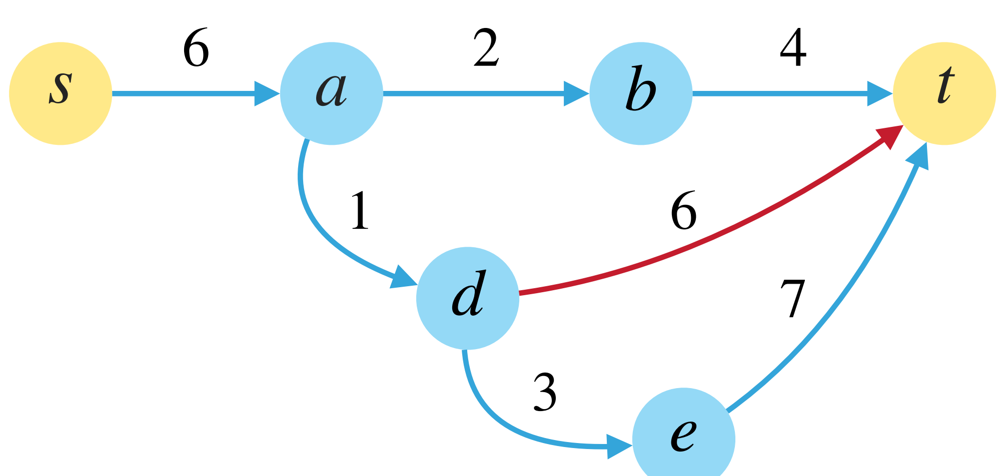
- $G = (V(G), E(G))$ – DAG
- $w : E(G) \to \mathbb{N}$ – cost-function
- $s \in V(G)$ – start vertex
- $t \in V(G)$ – target vertex
- $\beta \leq 1$ – agent's present-bias parameter
- r – reward the agent receives by reaching t



If for the agent in vertex v perceived cost $\zeta_M(P)$ exceeds $\beta \cdot r$, the agent abandons the whole project.

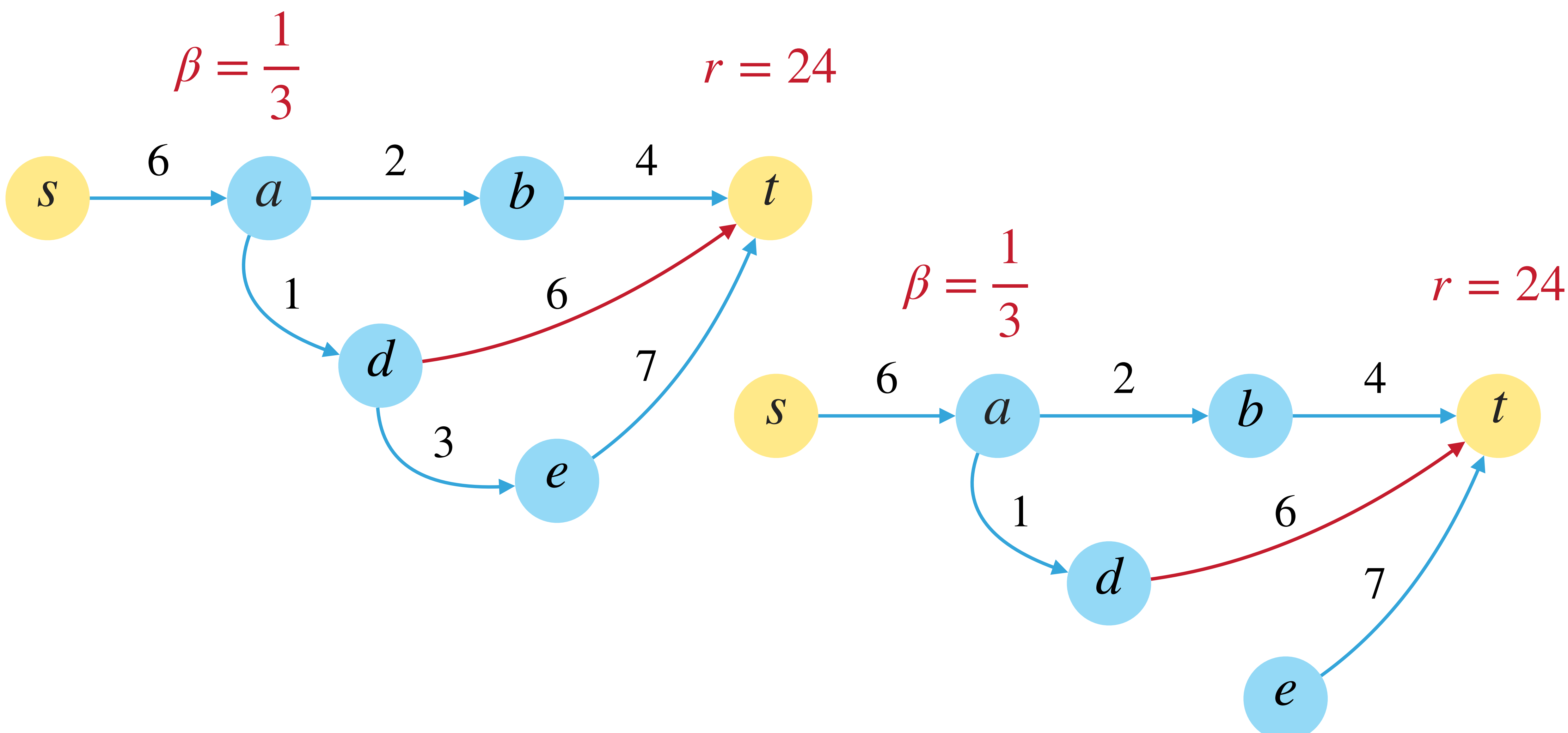
MOTIVATION

- Alice is PhD student.
- She has to accomplish several research projects to obtain her PhD.
- Bob is her advisor.
- Bob wants her to finish her study, he has additional interests too.
- The task corresponding to the arc dt is the most exciting part of the whole project.



EXAMPLE

- Bob can remove some tasks from Alice's plan.
- Bob decided to remove the arc de.



PROBLEMS

T-path-Deletion

Input: $M = (G, w, s, t, \beta, r)$, integer k and a set of arcs $T \subseteq E(G)$.

Task: Find a subset of arcs $D \subseteq E(G)$ of size at most k (or prove that no such set exists), such that after removing D from M, the agent will follow a T-path.

T-path-Addition

Input: $M = (G, w, s, t, \beta, r)$, integer k and a set of arcs $T \subseteq E(G)$, and a set of additional weighted arcs $A \subset V \times V$.

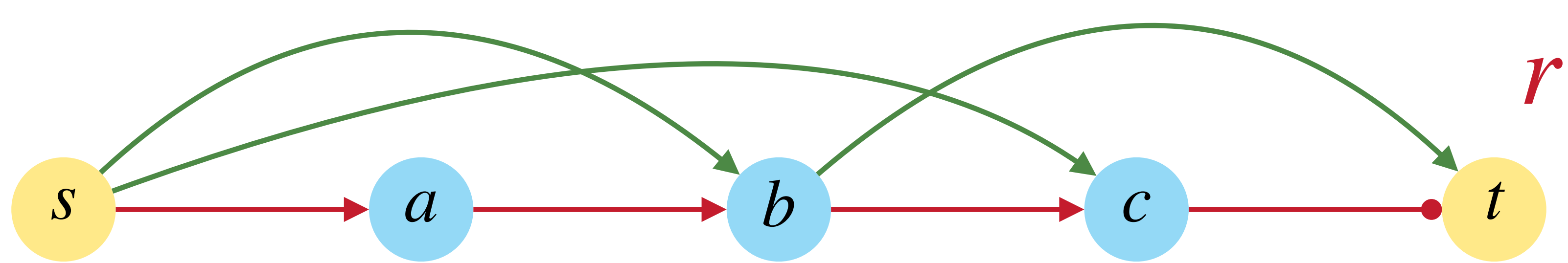
Task: Find a set S of at most k arcs from A (or prove that no such set exists), such that after adding these arcs to G the agent will follow a T-path.

RELATED PROBLEMS

- Finding a motivating subgraph (in our model T is empty)
- Tang, Teng, Wang and et al. show that this problem is NP-complete.
- Alberts and Kraft show approximation for reward.
- Fomin and Strømme studied parameterized complexity of computing a motivating subgraph.
- Oren and Soker studied P-motivating subgraph problem.
- Is there a subgraph of G, such that in this subgraph, the agent will follow along path P?
- In our model the prescribed arcs T form the edge set of P.

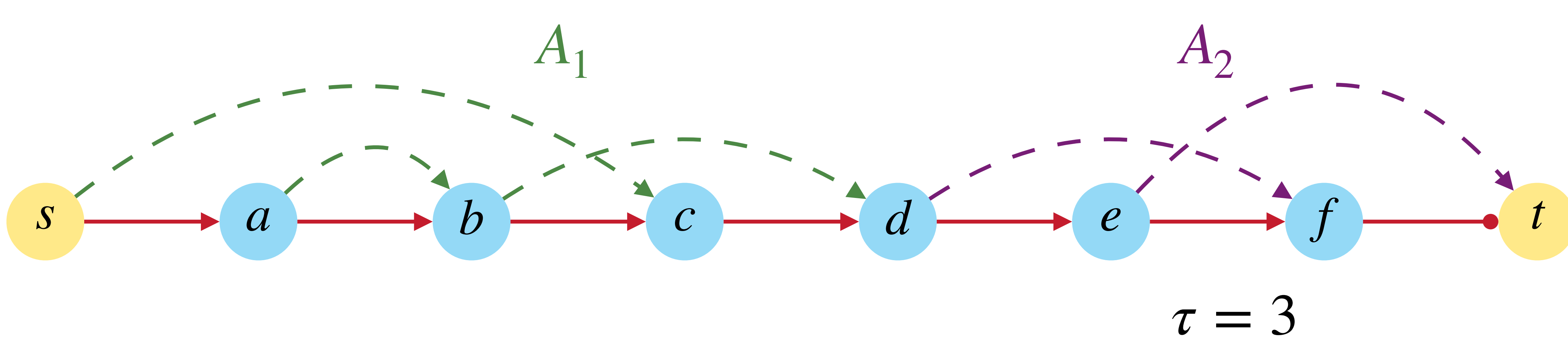
HARDNESS

- T-path-Deletion is W[1]-hard parameterized by k for any $\beta \leq 1$ even when T consists of a single arc.
- W[1]-hard parameterized by k for any $|T|$.
- T-path-Deletion is NP-hard with 2 different weights on the arcs, constant reward.
- T-path-Addition problem on the path with detours instances is NP-hard and W[1]-hard parameterized by k.

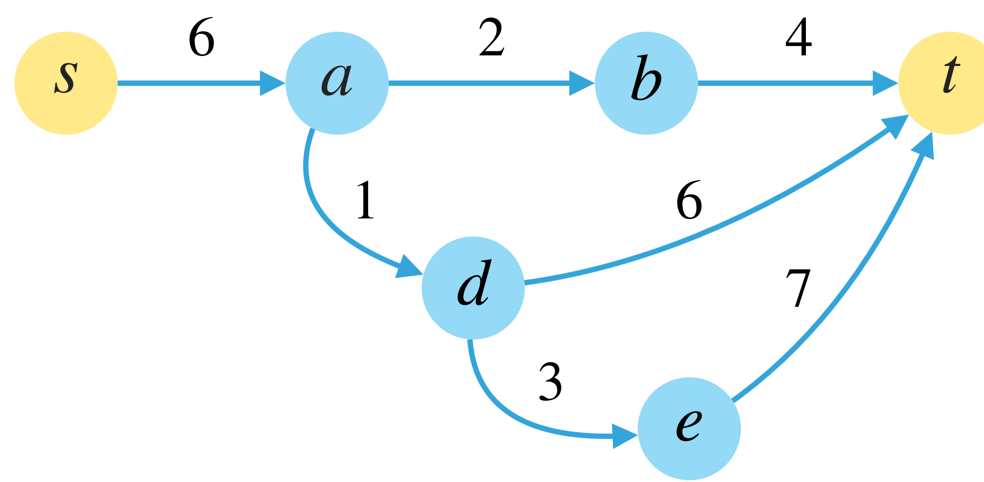


ALGORITHMS

- T-path-Deletion problem is solvable in time $O(m^{2k}) \cdot poly(n)$.
- T-path-Deletion admits a polynomial kernel parameterized by the size of a feedback edge set.
- T-path-Addition problem on paths with detours can be solved in time $2^{\tau} n^{O(1)}$, where τ is the size of maximum intersection component of A.



COST REDUCTION



- Other setting
- Alice has some budget.
- Now Bob wants to reduce Alice's costs to get a PhD.
- There is no reward anymore.

Optimal Reducing Expectation by Deletion/Addition (Opt-REC-Deletion/Addition)
Input: $M = (G, w, s, t, \beta)$, integer k.
Task: Find the minimum value of $\mathbf{E}(C_\beta)$, which can be achieved by removing (addition) no more than k arcs from the graph G.

RESULTS

- There is $(1/\beta)^n$-approximation.
- There is no FPT algorithm $(1/\beta - \varepsilon)^n$-approximation under W[1] $\neq$ FPT for any $\varepsilon \geq 0$.
- Hardness is similar to T-path-Deletion/Addition.
- Opt-REC-Addition problem on paths with detours can be solved in polynomial time.
- The process of adding and deleting arcs could be simulated by changing the weights of the arcs.
- In this case problem remains NP-hard.

OPEN PROBLEMS

- T-path problem with changing the weights of the arcs
- What happens to the complexity of the problems when you can put intermediate rewards to the vertices.
- Algorithms relative to other parameters.
- Kernel whose size is bounded by a size (even exponential) of a vertex cover of G.

PUBLICATIONS AND TALKS BASED ON THE RESULTS

- Inconsistent Planning: When in Doubt, Toss a Coin!
  Yuriy Dementiev, Fedor Fomin, Artur Ignatiev.
  AAAI 2022

- How to guide a present-biased agent through prescribed tasks?
  Tatiana Belova, Yuriy Dementiev, Fedor Fomin, Petr Golovach, Artur Ignatiev.
  Under review in ECAI 2024

- Euler Conference of small research groups

- Research seminar of the HSE Game Theory Laboratory