INCONSISTENT PLANNING: WHEN IN DOUBT, TOSS A COIN!

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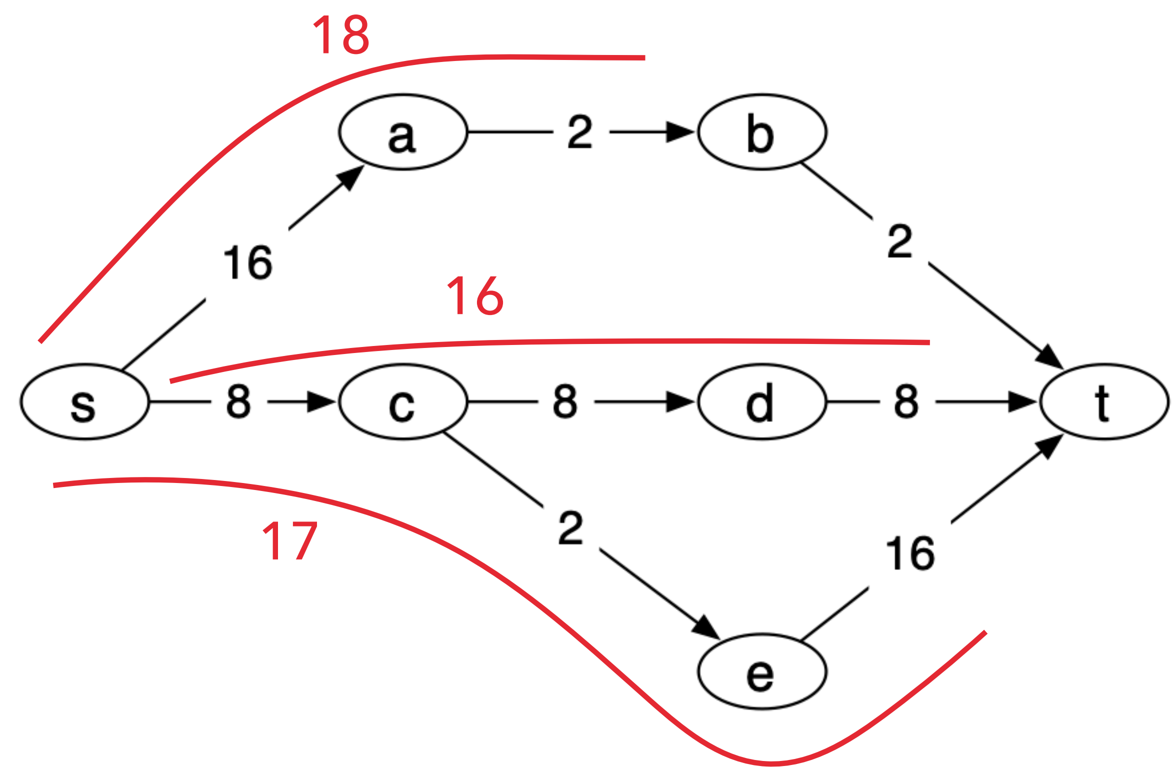
36th AAAI Conference on Artificial Intelligence
February 22 - March 1, 2022

PRESENT-BIAS PLANNING

- Behavioral Economic
- Study the impact of the gap between the anticipated costs of future actions and their real costs.
- Time-inconsistent planning: procrastination, abandonment, etc.
- **Akerlof (1991):** Graph theoretical model, where the cost of an action in the future is assumed to be β times smaller than its actual cost, for some $\beta < 1$.

KLEINBERG-OREN'S MODEL (EC 2014)

5-tuple $M = (G, w, s, t, \beta)$, where:



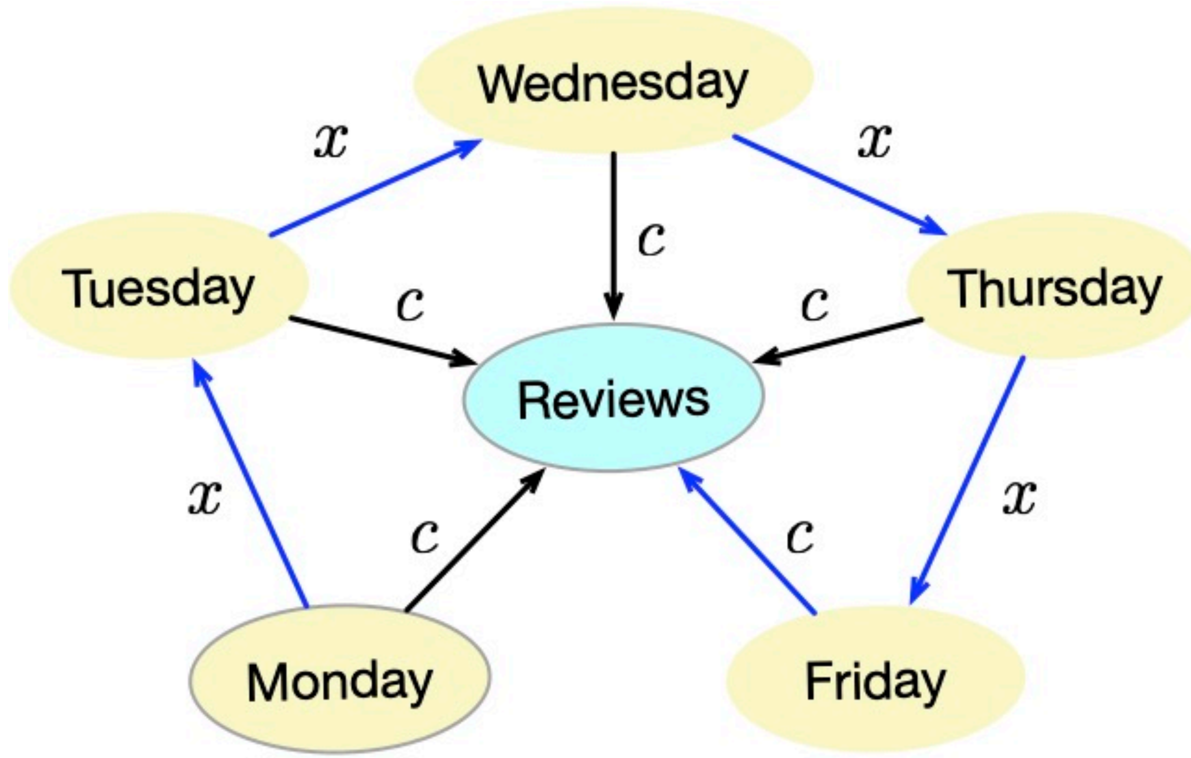
- $G = (V(G), E(G))$ – DAG
- $w : E(G) \to \mathbb{N}$ – cost-function
- $s \in V(G)$ – start vertex
- $t \in V(G)$ – target vertex
- $\beta \leq 1$ – agent's present-bias parameter.

In vertex v agent evaluates a path $P \subseteq G$ with edges $e_1, e_2, \ldots, e_p$ to cost $\zeta_M(P) = w(e_1) + \beta \cdot \sum_{i=2}^{p} w(e_i)$

OUR APPROACH

- Now when an agent is faced with multiple choices, he tosses a coin.
- The cost of the path traversed by the agent, $C_\beta(s, t)$, is a random variable.
- **Definition** (cost of irrationality)
  The cost of irrationality of the time-inconsistent planning model is $X_\beta = \dfrac{C_\beta(s,t)}{d(s,t)}$.

EXAMPLE



$$p_1 = p_2 = \frac{1}{2}$$

$$c = 6,\ x = 3,\ \beta = \frac{1}{2}$$

- $\Pr(X_\beta \leq 1) = \frac{1}{2}$
- $\Pr(X_\beta \leq 3/2) = \frac{1}{2} + \left(\frac{1}{2}\right)^2$
- $\Pr(X_\beta \leq 1 + (i-1)/2) = \sum_{j=1}^{i} \left(\frac{1}{2}\right)^j$ for $1 \leq i \leq 4$
- $\Pr(X_\beta \leq 3) = 1$

MODEL

- The instance of the time-inconsistent planning model is a $M = (G, w, s, t, p, \beta)$

- For each edge uv of the task graph, we assign the probability $p(u, v)$ of transition $u \to v$. For every $u \in V(G)$, $\sum_{uv \in E(G)} p(u, v) = 1$.

- The probability can be positive only for edges that could serve for transitions of the agent.

- Feasible path – s-t path P: $\Pr(\text{agent traverses } P) > 0$.

COMBINATORIAL RESULTS

- **Theorem**
  There is a family of graphs $\{G_n\}_{n=1}^{\infty}$ with an exponential number (in the number of vertices) of feasible paths of different costs.
- Supports a probabilistic tie breaking option.
- Difference between the costs of the minimum and maximum feasible paths in the graph can be exponential.

PROBLEMS

- **Estimating The Cost of Irrationality** (ECI)
  Input: $M = (G, w, s, t, p, \beta)$ and $W \geq 0$.
  Task: Compute $\Pr(X_\beta \leq W)$.

- **Minimum Cost of Irrationality** (MCI)
  Input: $M = (G, w, s, t, p, \beta)$.
  Task: Compute the minimum value W such that $\Pr(X_\beta \leq W) > 0$ and compute $\Pr(X_\beta \leq W)$.

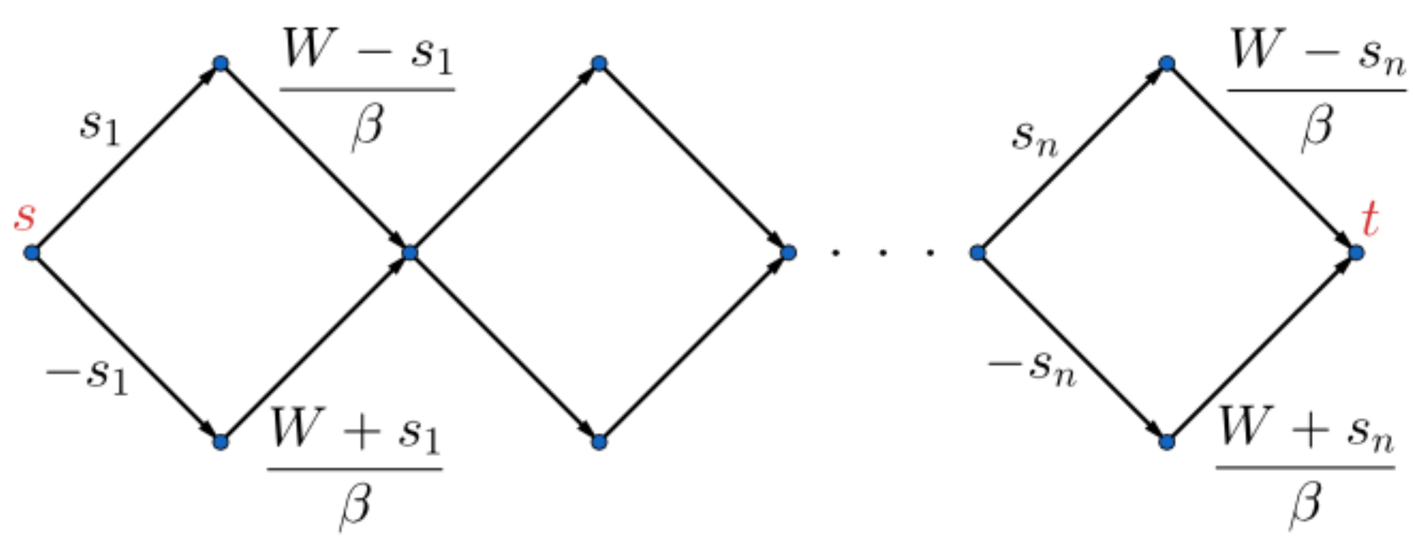
- **Maximum Cost of Irrationality**
  Input: $M = (G, w, s, t, p, \beta)$.
  Task: Compute the minimum value W such that $\Pr(X_\beta \leq W) = 1$.

COMPUTATIONAL RESULTS

- The ECI problem is #P-hard.

Counting Partitions → (Parsimonious reduction) → ECI

$$S = \{s_1, s_2, \ldots, s_n\}$$
$$S = S_1 \sqcup S_2$$



COMPUTATIONAL RESULTS

- The MCI problem admits an algorithm with running time $\mathcal{O}(n^3)$.
- The ECI problem admits an algorithm with running time $\mathcal{O}(\lfloor W \cdot d(s,t) \rfloor \cdot n^2 + n^3)$.
- The values $\mathbf{E}(X_\beta)$ and $\mathbf{Var}(X_\beta)$ are computable in time $\mathcal{O}(n^3)$.
- Using Chebyshev's inequality:
$$\Pr(\,|\,C_\beta - \mathbf{E}(C_\beta)\,| \leq 2\sqrt{\mathbf{Var}(C_\beta)}) \geq \frac{3}{4}$$

PARAMETERIZED COMPLEXITY

- Parameterized problem – $Q \subseteq \Sigma^* \times \mathbb{N}$. Input of Q is a pair (I, k), where k is the parameter of the problem.
- Q is FPT if it can be decided whether $(I, k) \in Q$ in time $f(k) \cdot |I|^{\mathcal{O}(1)}$.
- The W-hierarchy is a collection of computational complexity classes: FPT = W[0] ⊆ W[1] ⊆ W[2] ⊆ … .
- It is widely believed that FPT ≠ W[1].

PARAMETERIZED RESULTS

- The ECI problem is W$[1]$-hard parameterized by $\mathrm{vc}(G)$ and by $\mathrm{fvs}(G)$.
- The ECI problem admits an algorithm of running time $n^{\mathcal{O}(\mathrm{fvs}(G))} \cdot \mathrm{fvs}(G)^{\mathrm{fvs}(G)}$.
- The ECI problem is solvable in time $2^{\mathrm{fes}(G)} \cdot poly(n)$.

OPEN PROBLEMS

- We gave a polynomial time algorithm computing $\mathbf{E}(X_\beta)$. In what class of complexity does the following problem lie: delete at most k edges (or vertices) such that in the resulting graph the expected cost of irrationality is less than $\mathbf{E}(X_\beta)$?