SEVERAL STORIES ABOUT HIGH-MULTIPLICITY EFX ALLOCATION

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SETTING

- N is a set of n agents.
- M is a set of m goods that cannot be divided or shared.
- Each agent $i \in N$ is equipped with an additive valuation function $v_i : 2^M \to \mathbb{N}_{\geq 0}$, which assigns a non-negative integer
- $v_i(S) = \sum v_i(g)$ for any subset of items $S \subseteq M$.
- Item type is a vector of length n, where the i-th coordinate is the value of the good's utility for the i-th agent.

FAIRNESS AND EFFICIENCY

An allocation A is envy-free up to any item (EFx) if it satisfies:

$$\forall i, j \in N: \; u_i(A_i) + \min_{z \in A_j} u_i(z) \geq u_i(A_j) \,.$$

An allocation A is pareto-optimal (PO) if there is no other allocation B such that:

$$\begin{cases} \forall \, i \in N: \; u_i(B_i) \geq u_i(A_i), \\ \exists \, j \in N: \; u_j(B_j) > u_j(A_j) \,. \end{cases}$$

MOTIVATION

- State-of-the-art approaches are too slow
- What if we change parameters?
- We choose the number of EFx allocations as an additional parameter

RESULTS

- The problem of existence of an $\mathrm{EFx} + \mathrm{PO}$ allocation is NP-hard even for two agents.

- The problem of searching for $\mathrm{EFx} + \mathrm{PO}$ allocation allows for an FPT -algorithm based on the number of agents n, the number of EFx allocations s, and the number of item types k. Its running time is:

$$O^*((2snk + 4kn^2)^{5snk+10kn^2}s)$$

- The total number of EFx allocations in the problem with m objects and two agents does not exceed $(\lceil \frac{m+k}{k} \rceil)^k$, where k is the number of different types of items.

- In practice number of EFx allocations is lower.