Inconsistent Planning: When in doubt, toss a coin!

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Introduction

Time-inconsistent behavior is the term in behavioral economics and psychology describing the behavior of an agent optimizing a course of future actions but changing his optimal plans in the short run without new circumstances [1].

A standard assumption in behavioral economics used to explain the gap between long-term intention and short-term decision-making is the notion of *present bias*. According to [2], when considering trade-offs between two future moments, present-biased preferences give stronger relative weight to the earlier moment as it gets closer.

A simple mathematical model of present bias was suggested in [3]. In Akerlof's model, the salience factor causes the agent to put more weight on immediate events than on the future.

Kleinberg and Oren [4, 5] introduced an elegant graph-theoretic model encapsulating the salience factor and scenarios of Akerlof. The approach is based on analyzing how an agent traverses from a source s to a target t in a directed edge-weighted graph G.

New model

An instance of the *time-inconsistent planning model* is a 6-tuple $M = (G, w, s, t, p, \beta)$ where:

1. $G = (V(G), E(G))$ is a directed acyclic n-vertex graph called a *task graph*.
2. $w : E(G) \to \mathbb{N}$ is a function representing the costs of transitions between states. The transition of the agent from state u to state v along arc $uv \in E(G)$ is of cost $w(uv)$.
3. The agent starts from the start vertex $s \in V(G)$, $t \in V(G)$ is the target vertex.
4. For each edge uv of the task graph, we assign the probability $p(u, v)$ of transition $u \to v$. For every $u \in V(G)$, $\sum_{uv \in E(G)} p(u, v) = 1$. Moreover, the probability can be positive only for edges that could serve for transitions of the agent.
5. $\beta \le 1$ is the agent's present-bias parameter.

Agent actions in the model:

- An agent is initially at vertex s and his task is to reach the target t.
- When standing at a vertex v, the agent evaluates all possible paths from v to t: v-t path $P \subseteq G$ with edges $e_1, e_2, \dots, e_p$ is evaluated to *perceived* cost

$$\zeta_M(P) = w(e_1) + \beta \cdot \sum_{i=2}^{p} w(e_i).$$

- For a vertex v, its *perceived cost to the target* is the minimum perceived cost of any path to t,

$$\zeta_M(v) = \min\{\zeta_M(P) \mid P \text{ is a } v\text{-}t \text{ path}\}.$$

- Then the agent picks the first edge of one of the *perceived paths* (with perceived cost $\zeta_M(v)$), according to the distribution p, and traverses this edge, say vu.
- At the vertex u, the agent repeats the procedure and so moves on until he reaches the vertex t.

We can define the cost of agent's path with present-bias β as discrete random variable C_β with $\Pr(C_\beta = W)$ being the probability that the path traversed by the agent is of cost W. The *cost of the irrationality* of the time-inconsistent planning model $M = (G, w, s, t, p, \beta)$ is

$$X_\beta = \frac{C_\beta}{d(s,t)}.$$

Questions

The Estimating the Cost of Irrationality (ECI) problem

- Input: A time-inconsistent planning model $M = (G, w, s, t, p, \beta)$, and $W \ge 0$.
- Task: Compute $\Pr(X_\beta \le W)$.

The Minimum Cost of Irrationality (MCI) problem

- Input: A time-inconsistent planning model $M = (G, w, s, t, p, \beta)$.
- Task: Compute the minimum value W such that $\Pr(X_\beta \le W) > 0$ and compute $\Pr(X_\beta \le W)$.

Similarly, you can define the Maximum Cost of Irrationality problem, where it is necessary to compute the minimum value W such that $\Pr(X_\beta \le W) = 1$.

Example

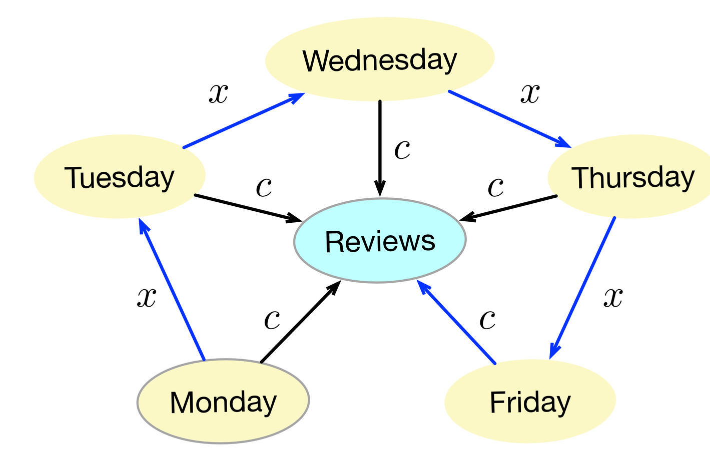

Figure 1: Task graph G with the source point $s = \texttt{Monday}$ and the target point $t = \texttt{Reviews}$.

Let $c = 6$, $x = 3$, $\beta = \frac{1}{2}$. Bob does not have preferences between two actions of minimum perceived costs and thus pursue one of the actions with probability $p = 1/2$.

- $d(s,t) = c = 6$, $\Pr(C_\beta \le 6) = \frac{1}{2}$, $\Pr(X_\beta \le 1) = \frac{1}{2}$
- $\Pr(X_\beta \le 9/6 = 3/2) = \frac{1}{2} + \left(\frac{1}{2}\right)^2$.
- $1 \le i \le 4$, $\Pr(X_\beta \le 1 + (i-1)/2) = \sum_{j=1}^{i} \left(\frac{1}{2}\right)^j$.
- $\Pr(X_\beta \le 3) = 1$.

Lower Bound

Theorem

The ECI problem is #P-hard and W[1]-hard parameterized by $\mathrm{vc}(G)$ and by $\mathrm{fvs}(G)$.

To obtain prove #P-hard and W[1]-hardness of ECI, we reduce from Counting Partitions and Modified k-Sum.

The Counting Partitions problem

- Input: Set of positive integers $S = \{s_1, \dots, s_n\}$.
- Task: Count the number of partitions of S into sets S_1 and S_2 such that the sums of numbers in both sets are equal.

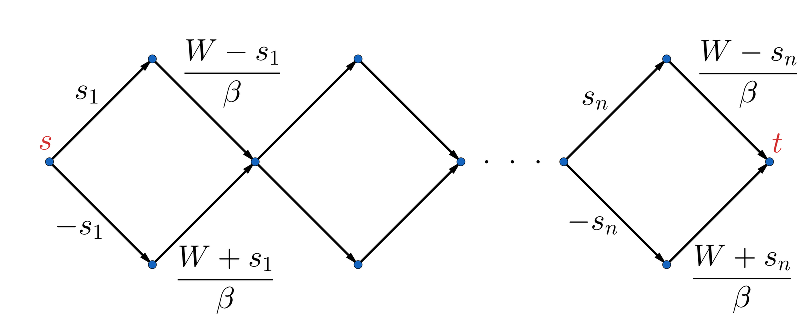

Figure 2: Gadget used in #P-hardness proof.

Sketch of the reduction:

- All paths in the graph are feasible for the agent.
- Partition of $S \longleftrightarrow$ agent's path of cost $\frac{n \cdot W}{\beta}$.

The Modified k-Sum problem

- Input: Sets of integers $X_1, X_2, \dots, X_k$ and integer T.
- Parameter: k.
- Task: Decide whether there is $x_1 \in X_1, x_2 \in X_2, \dots, x_k \in X_k$ such that $x_1 + \ldots + x_k = T$.

Algorithms

Theorem

- Minimum Cost of Irrationality and Maximum Cost of Irrationality admits an algorithm with running time $\mathcal{O}(n^3)$.
- ECI admits an algorithm with running time $\mathcal{O}(\lfloor W \cdot d(s,t) \rfloor \cdot n^2 + n^3)$.
- $\mathbf{E}(X_\beta)$ and $\mathbf{Var}(X_\beta)$ are computable in time $\mathcal{O}(n^3)$.
- ECI in XP parameterized by $\mathrm{fvs}(G)$.
- ECI is FPT parameterized by $\mathrm{fes}(G)$.

Sketch of the algorithm:

- Preprocessing: topological sorting of vertices, as well as the calculation of the shortest paths between all pairs of vertices in a graph.
- Dynamic programming in the direction from s to t. Precomputed shortest distances are used to model the agent's decisions at the vertices in linear time.
- For the MCI problem, the following values are calculated at each vertex v: the cost of the agent's shortest path from s to v, and probability with which the agent will come to vertex v along the minimum cost path.
- For the ECI problem, at each vertex v we calculate an array of size $\lfloor W \cdot d(s,t) \rfloor$, where the position i is the probability that the agent came to the vertex v along the path of cost i.
- To obtain parametrized algorithms for the ECI problem, we estimate the number of different s-t paths in the graph as functions of the parameters.

Open questions

The Reducing the Mathematical Expectation of Irrationality problem

- Input: A time-inconsistent planning model $M = (G, w, s, t, p, \beta)$, and integer k.
- Task: Is there a set $S \subseteq E$, $|S| \le k$ such that in $G - S$ the mathematical expectation of C_β has decreased.

Of course, there is a brute-force algorithm solving the problem in time $n^{\mathcal{O}(k)}$ by calling our polynomial-time algorithm for each of the $\binom{n}{k}$ possibilities of deleting k edges (or vertices). But whether the problem is FPT parameterized by k, is an interesting open question.

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